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Whence $2az=z^2+a^2-2ax$ ....(2).

Also from the figure, $GM/LM=\frac{1}{2}z/y=a/z$, from which $z=\surd(2ay)$.........(3).

Substitute this value of z in (2), square and arrange terms, and there results,

$$4x^2-8xy+4y^2-4ax-4ay+a^2=0 \text{ .... (4).}$$

Equation (4) is that of a parabola, tangent to the axes at a distance of $\frac{1}{2}a$ from the origin.

59. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Find the radius of a sphere of given specific gravity which will rest just immersed in a fluid whose density varies as its depth.

I. Solution by **HENRY HEATON, M. Sc.,** Atlantic, Iowa; **E. L. SHERWOOD, A. M.,** Superintendent of Schools, West Point, Miss., and **W. W. LANDIS, A. M.,** Professor of Mathematics, Dickinson College, Carlisle, Pa.

Put r=radius of sphere, s_1=its specific gravity, s_2=specific gravity of the fluid at the depth of 1 foot, and $a=62.5$ pounds=weight of a cubic foot of water. The weight of the sphere is $\frac{4}{3}as_1\pi r^3$. The weight of the sphere must equal the weight of the fluid displaced. To find this, put x=the distance of a horizontal section of the sphere below the surface. The weight of an equivalent quantity of the fluid between two sections whose distance apart is dx is $as_2\pi(r^2-x^2)xdx$. Hence the weight of fluid displaced is

$$as_2\pi\int_0^{2r}(r^2-x^2)xdx=as_2\pi r^4=\tfrac{4}{3}as_1\pi r^3. \quad \therefore\ r=s_1/s_2.$$

II. Solution by **ALFRED HUME, C. E., D. Sc.,** Professor of Mathematics, University of Mississippi, University, Miss.; and **J. C. NAGLE. M. A., M. C. E.,** Professor of Civil Engineering, in State Agricultural and Mechanical College, College Station, Texas.

Let the origin of coördinates be taken at the lowest point of the sphere, the axis of x vertical, that of y horizontal. The equation of the section of the sphere made by the xy-plane is $y^2=2Rx-x^2$, R being the sphere's radius.

If ρ be the density of the liquid at a depth of unity, then at a distance x above the origin, the density is $\rho(2R-x)$. The weight of the displaced liquid is

$$\int_0^{2R}\pi y^2 dx.\rho(2R-x)g.$$

If c be the density of the sphere, its weight is $\frac{4}{3}\pi R^3cg$.

These being equal

$$\tfrac{4}{3}cR^3=\rho\int(2Rx-x^2)(2R-x)dx,$$

the limits being $2R$ and 0, $=\frac{4}{3}\rho R^4$; $R=c/\rho$.

III. Solution by the **PROPOSER.**

Let δ, δ' be the respective densities of the sphere and of the fluid at a unit depth, and take the upper extremity of the vertical diameter of the sphere for origin, that diameter being in the axis of x.

$\delta' x$=the density of the fluid at depth x, and for equilibrium the mass of sphere immersed equals the mass of fluid displaced.

Then, r being the radius of the sphere,

$$\delta.\tfrac{4}{3}\pi r^3=\int\int \pi y^2 dx.\delta x=\int_0^{2r} \pi\delta' x(2rx-x^2)dx=\delta'\tfrac{4}{3}\pi r^4. \quad \therefore\ r=\delta/\delta'.$$

Also solved by *J. SCHEFFER* and *G. B. M. ZERR*.

60. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

What must be the ratio of the two legs of a uniform and heavy right triangle suspended from the center of the inscribed circle, if this triangle will rest with the shorter leg in a horizontal position ?

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If a and b be the longer and shorter side about the right angles, and r the radius of the inscribed circle, we should have

$$\tfrac{1}{2}r[a+b+\sqrt{(a^2+b^2)}]=\tfrac{1}{2}ab \text{ .... }(1),$$

$$\text{or } r=\tfrac{1}{2}a+\tfrac{1}{2}b-\tfrac{1}{2}\sqrt{(a^2+b^2)} \text{ .... }(2).$$

The triangle is kept in equilibrium by the action of the weights of the sides vertically downwards and the reaction of the point of support.

Taking moments about this fallen point,

$$ra=(\tfrac{1}{2}b-r)\sqrt{(a^2+b^2)}+b(\tfrac{1}{2}b-r) \text{ .... }(3),$$

the sides being taken proportional to their weights.

r from (2) put into (3) gives, by reduction,

$$3a=4b \text{ or } a=\tfrac{4}{3}b \text{ .... }(4).$$

II. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and W.W. LANDIS, A. M., Professor of Mathematics, Dickinson College. Carlisle, Pa.

If a and b represent the legs of the triangle the distance of the center of gravity from the leg a is $\frac{1}{3}b$. The radius of the inscribed circle is

$$r=\tfrac{1}{2}[a+b-\sqrt{(a^2+b^2)}].$$

To secure equilibrium in the desired position r must equal $\frac{1}{3}b$, or

$$\tfrac{1}{3}b=\tfrac{1}{2}[a+b-\sqrt{(a^2+b^2)}].$$

Whence $b=\frac{3}{4}a$, or $b/a=\frac{3}{4}$.

In this triangle the centee of gravity is directly above the center of the inscribed circle. Hence the equilibrium is unstable.

III. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va., and J. SCHEFFER, A. M., Hagerstown, Md.

In order that AB may be horizontal the point of suspension O and the center of gravity G must be in the same straight line perpendicular to AB.

Let $AB=x$, $BC=mx$, $AC=x\sqrt{(1+m^2)}$. Then $HB=r$=radius of in-circle.